Leveraging Sparsity in Distribution Grids

System Identification and Harmonic State Estimation

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6 December, 2018

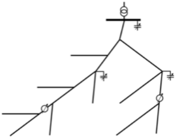
ePerf Workshop

Outline

Tremendous Changes in the Last Mile

Central operator model

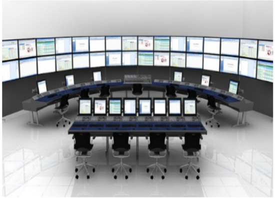
reactive, conservative w/ ad-hoc interventions

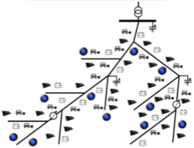


Drivers of change

Transition model

proactive, distributed and participative





New Planning and Operation Paradigm

Distribution utilities have to

- Run power flow analysis $\rightarrow$ planning
- Control distributed energy resources (DER) $\rightarrow$ operation
- Detect, classify, and localize events shortly after their occurrence $\rightarrow$ operation
- Locate harmonic sources and estimate the distribution of harmonic voltages $\rightarrow$ operation

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The system identification, event detection, state estimation problems that must be solved are usually ill-posed

Sparsity Can Help...

- Distribution networks (radial or non-radial) have sparse topology
- High-precision sensors (D-PMUs and harmonic meters) are relatively sparse compared with the number of nodes
- Events are sparse in time and frequency domains
  - tap changes
  - switching operations
  - harmonics
  - faults

Learning a Sparse Model from Data

Distribution network topology is necessary for several analytics and control applications, yet it is typically unknown

- Situational awareness
- Control applications (model-based)
- State estimation
- Cybersecurity

Goal

a) estimate the admittance matrix of a distribution network from measurements of voltage and current magnitudes and phase angles
b) track changes in the identified model in near real-time

Problem Formulation

$$\underbrace{\begin{bmatrix} I_1(1\cdots K) \\ I_2(1\cdots K) \\ \vdots \\ I_N(1\cdots K) \end{bmatrix}}_{I_{\text{bus}}^K} = \underbrace{\begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1N} \\ Y_{12}^\top & Y_{22} & \dots & Y_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{1N}^\top & Y_{2N}^\top & \dots & Y_{NN} \end{bmatrix}}_{Y_{\text{bus}}} \underbrace{\begin{bmatrix} V_1(1\cdots K) \\ V_2(1\cdots K) \\ \vdots \\ V_N(1\cdots K) \end{bmatrix}}_{V_{\text{bus}}^K},$$

Each off-diagonal block of Y_{bus} is a submatrix $Y_{mn} = -Z_{mn}^{-1}$ corresponding to the admittance of a multi-phase line, and each diagonal block is a submatrix

$$Y_{nn} = \sum_{m \in \{o | (o,n) \in \mathcal{E}\}} \left(\frac{1}{2} Y_{mn}^s + Z_{mn}^{-1} \right).$$

Problem Formulation

Estimate Y_{bus} (or at least a large part of it) given V_{bus}^{K} and I_{bus}^{K} for time slots: $1 \cdots K$

Key observations

- Y_{bus} is symmetric and sparse
- V_{bus} is low rank
- measurements are noisy
- only a small number of nodes are monitored

Base Case: V_{bus} is full rank, network is fully observed

$$\widehat{Y}_{\text{bus}} = \underset{Y}{\text{argmin}} \left\| Y V_{\text{bus}}^K - I_{\text{bus}}^K \right\|_F \quad \text{subject to} \quad Y \in \mathbb{S}^{N \times N}.$$

Enforcing sparsity:

$$\hat{Y}_{\text{bus}} = \underset{Y}{\text{argmin}} \left\| ((V_{\text{bus}}^K)^\top \otimes \mathbb{1}_N)\text{vec}(Y) - \text{vec}(I_{\text{bus}}^K) \right\|_2 \quad \text{subject to} \quad Y \in \mathbb{S}^{N \times N}, \quad \|\text{vec}(Y)\|_0 \le \delta,$$

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We can write $\text{vec}(Y) = Q_Y f(Y)$ where $f(Y)$ collects the lower triangular elements of Y and Q_Y is a unique binary matrix that converts $f(Y)$ to $\text{vec}(Y)$

$$f(\widehat{Y}) = \underset{x \in \mathbb{C}^{(N^2+N)/2 \times 1}}{\operatorname{argmin}} \left\| \left({V_{\text{bus}}^{K}}^\top \otimes \mathbb{1}_N \right) Q_Y x - \text{vec}(I_{\text{bus}}^{K}) \right\|_2^2 + \lambda \|x\|_0,$$

Lasso

We solve the relaxed problem

$$\min_{x \in \mathbb{C}^{(N^2+N)/2 \times 1}} \left\| \underbrace{\left({V_{\text{bus}}^K}^\top \otimes \mathbb{1}_N \right) Q_Y}_{A} x - \underbrace{\text{vec}(I_{\text{bus}}^K)}_{b} \right\|_2^2 + \lambda \left\| x \right\|_1 .$$

The lasso shrinks the elements of Y_{bus} toward 0 as λ increases

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The lasso shrinks the elements of Y_{bus} toward 0 as λ increases

Elements of Y do not have the same scales

Adaptive Lasso

The adaptive lasso applies less shrinkage whenever the true unknown variable is large

$$\min_{x \in \mathbb{C}^{(N^2+N)/2 \times 1}} \|Ax - b\|_2^2 + \lambda \sum_i \frac{|x_i|}{|\hat{x}_i|^{\gamma}},$$

where γ is a positive parameter and $\hat{x}_i$ is an initial estimator for x_i, e.g., the ordinary least squares (OLS) estimator defined:

$$\hat{x} = (A^\top A)^{-1} A^\top b.$$

Low Rank Structure of Distribution Networks

V_{bus} is low rank in practice

Can we still recover some part of the admittance matrix?

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Can we still recover some part of the admittance matrix?

$$\begin{bmatrix} \mathbb{I}_1 \\ \mathbb{I}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbb{Y}_{1,1} & \mathbb{Y}_{1,2} \\ \mathbb{Y}_{1,2}^\top & \mathbb{Y}_{2,2} \end{bmatrix}}_{\mathbb{Y}} \begin{bmatrix} X\mathbb{V}_2 \\ \mathbb{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbb{Y}_{1,1}X + \mathbb{Y}_{1,2} \\ \mathbb{Y}_{1,2}^\top X + \mathbb{Y}_{2,2} \end{bmatrix} \mathbb{V}_2$$

$$-\left(X^\top \otimes X^\top\right) \text{vec}(\mathbb{Y}_{1,1}) + \text{vec}(\mathbb{Y}_{2,2}) = \text{vec}(C).$$

$$\begin{bmatrix} f(\widehat{\mathbb{Y}}_{1,1}) \\ f(\widehat{\mathbb{Y}}_{2,2}) \end{bmatrix} = \arg\min_x \lambda \sum_i w_i |x_i| + \left\| \left[-(X^\top \otimes X^\top) Q_{Y_{11}},\, Q_{Y_{22}}\right] x - \text{vec}(C) \right\|_2^2 .$$

Partial Observability

What if only a small number of nodes are equipped with sensors?

Depends on the degree of nodes that are not monitored and how they are connected to the rest of the network (work in progress)

Early Event Detection

Assumption

Only a small number of elements of Y would change simultaneously (or in a short period of time)

$$\begin{aligned} \min_{\Delta Y \in \mathbb{S}^{\sum_{n\in\mathcal{N}} |\mathcal{P}_n| \times \sum_{n\in\mathcal{N}} |\mathcal{P}_n|}} \quad & \|\text{vec}(\Delta Y)\|_0 \\ \text{subject to} \quad & I_{\text{bus}}^{t\to t+K} - Y_{\text{bus}}^0 V_{\text{bus}}^{t\to t+K} = \Delta Y V_{\text{bus}}^{t\to t+K} \end{aligned}$$

can be relaxed and converted to a weighted regularized ℓ_1-norm optimization

Background

The growing adoption of power electronic devices and large non-linear loads has increased *harmonic-related* power quality problems

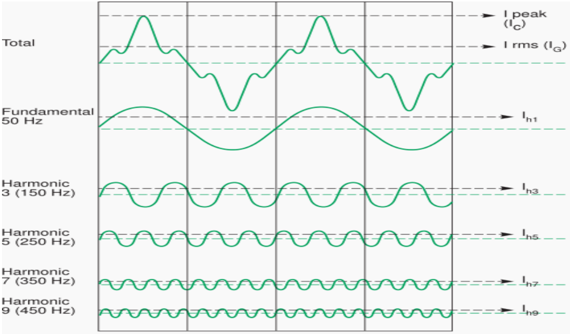
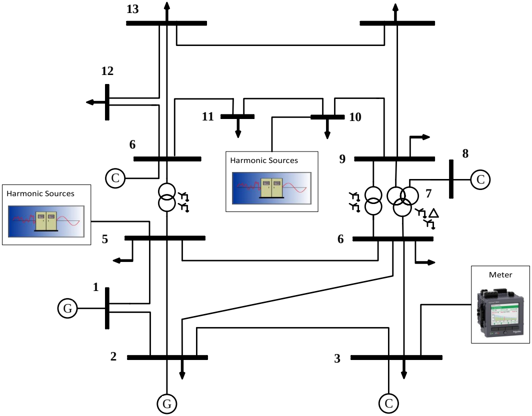


Figure: Harmonic currents.

Harmonic State Estimation (HSE)

- Locate harmonic sources
- Estimate harmonic voltage distribution



Problem formulation

HSE aims to estimate state variables (injected currents by harmonic sources), x, from harmonic measurements, z, given the measurement noise, ξ:

$$z(h) = \Phi(h)x(h) + \xi,$$

$$z(h) = \begin{bmatrix} V_{L(1)}(h) \\ \vdots \\ V_{L(\kappa_1)}(h) \\ I_{L(1)}(h) \\ \vdots \\ I_{L(\kappa_2)}(h) \end{bmatrix}, \Phi(h) = \begin{bmatrix} a_{L(1)1} & \cdots & a_{L(1)\bar{N}} \\ \vdots & \ddots & \vdots \\ a_{L(\kappa_1)1} & \cdots & a_{L(\kappa_1)\bar{N}} \\ b_{L(1)1} & \cdots & b_{L(1)\bar{N}} \\ \vdots & \ddots & \vdots \\ b_{L(\kappa_2)1} & \cdots & b_{L(\kappa_2)\bar{N}} \end{bmatrix},$$

where $\Phi(h)$ is a known (or estimated) *system matrix* with $a_{L(i)j} = [Y^H(h)^{-1}]_{L(i)W(j)}$ and $b_{L(i)j} = [Y^{bf}(h)Y^H(h)^{-1}]_{L(i)W(j)}$

Problem formulation

Sparsity

The state variable is sparse when there is a small number of sources producing harmonics simultaneously at each harmonic order:

$$||x(h)||_0 \leq s$$

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ℓ_0 Problem

Taking sparsity of $x(h)$ into account, the HSE problem can be formulated as:

$$\begin{aligned} \min \quad & ||x(h)||_0 \\ \text{s.t.} \quad & ||z(h) - \Phi(h)x(h)||_2 \leq \eta \end{aligned}$$

It is a non-trivial problem!

An ℓ_1 minimization is solved as a convex relaxation of this problem

Challenges:

- limited measurements pose an under-determined equation
- strong correlation between columns of $\Phi(h)$ (high coherence)

The Proposed Harmonic State Estimator

SBL-based State Estimator

$$\hat{x}^{(k)} = \arg\min_{x} \frac{1}{2}||\tilde{z} - \tilde{\Phi}x||_2^2 + \lambda \sum_{i=1}^{2\bar{N}} u_i^{(k)}|x_i|, \quad (1)$$

$$\gamma_i^{(k)} = \hat{x}_i^{(k)}/u_i^{(k)}, \quad (2)$$

$$u_i^{(k+1)} = [\tilde{\Phi}_{\cdot i}^{\top}(\lambda I + \tilde{\Phi}\Gamma^{(k)}\tilde{\Phi}^{\top})^{-1}\tilde{\Phi}_{\cdot i}]^{\frac{1}{2}}, \quad (3)$$

re-weighting parameter, u_i, promotes the sparsity of x
weight parameter, λ, trades off sparsity and estimation error.

SBL can effectively deal with high coherence of the system matrix!

Observability analysis

Definition

A power system is s-observable if the state variables satisfying the sparsity condition $||x||_0 \leq s$ can be determined uniquely given harmonic measurements z.

Lemma

Sufficient Condition: A power system is s-observable if $Spark(\Phi) > 2s$.

Noise-Free Case

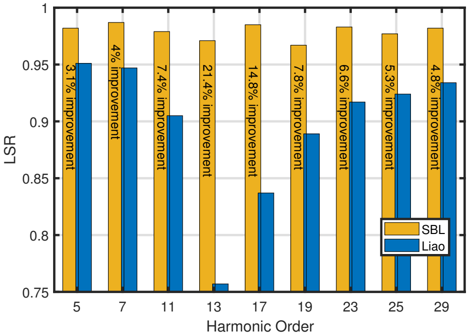

Figure: Comparing LSR of Lasso and SBL for different harmonic order under $\mathbb{M}_a$. The LSR increased by 8.3% on average

Noise-Free Case

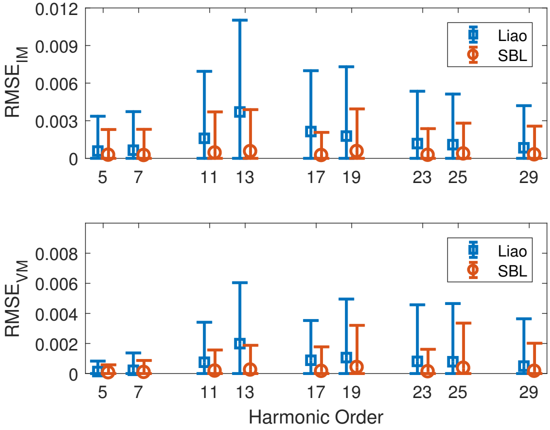

Figure: Comparing RMSE of Lasso with that of SBL

Summary

Takeaways

- The proposed SBL-based harmonic state estimator has superior performance despite the strong correlation between columns of the system matrix; this eliminates the need to check the *restricted isometry property* (RIP) or coherence condition.
- The proposed state estimator outperforms the state-of-the-art in terms of estimation and localization errors using limited measurements.

Summary

Takeaways

- The proposed SBL-based harmonic state estimator has superior performance despite the strong correlation between columns of the system matrix; this eliminates the need to check the *restricted isometry property* (RIP) or coherence condition.
- The proposed state estimator outperforms the state-of-the-art in terms of estimation and localization errors using limited measurements.

Next steps

- We are studying the optimal placement of harmonic meters.
- We are using smart meter and PMU data to estimate the system matrix in real-time.

Questions?

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